

# THE UNIVERSITY OF AUCKLAND

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EXAMINATION FOR BA BSc ETC 1999

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## COMPUTER SCIENCE

### Intelligent Active Vision

(Time allowed: TWO hours)

NOTE: Answer questions A,B,C,D, and either questions E and F  
or questions G and H  
Put the answers in the boxes below the questions.  
Use of calculators is *permitted*  
This is an *open* book exam

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SURNAME:

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FORENAME(S):

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STUDENT ID:

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LOGIN:

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TOTAL MARKS (OUT OF 100):

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### Marks:

A	B	C	D	E or G	F or H	Total
10	5	25	20	30	10	100

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## A Camera Calibration (Small and Medium Groups)

1. A sequence  $T$  of **translation (i.e. shifts)** transformations is applied to a **2 dimensional** cartesian coordinate system.

Given the matrix representation in homogenous coordinates, the coordinates of a point in the new coordinate system  $v^* = [X^*, Y^*]^T$  can be obtained from the old coordinates  $v = [X, Y]^T$  by the formula  $v^* = Av$ , where  $A$  is the 3x3 transformation matrix.

Show the transformation matrix for translations. Use variables  $u_1, u_2, u_3, \dots$  for unknown values.

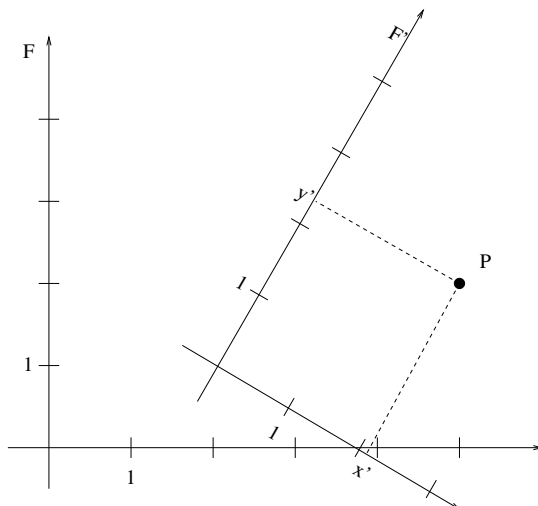
[2 marks]

$$A = \begin{bmatrix} 1 & 0 & u_1 \\ 0 & 1 & u_2 \\ 0 & 0 & 1 \end{bmatrix}$$

How many point pairs  $(v^*, v)$  are necessary to uniquely determine the transformation matrix  $A$ ? [1 mark]

We have two unknown variables. Each point gives us two equations. Therefore, we need only one points. Any point will do.

2. There are two coordinate systems,  $F$  and a new coordinate system  $F'$ .  $F'$  is obtained from  $F$  by a clockwise rotation by  $30^\circ$  and a translation of the origin to point  $(2, 1)$ .



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QUESTION/ANSWER SHEET

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Compute the coordinates ( $P' = (x', y')$ ) of point  $P = (5, 2)$  in the new coordinate system  $F'$ . [7 marks]

$$\text{Vector } O' \text{ to P is } \begin{bmatrix} 5 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\text{Angle } \alpha_F = \tan^{-1}(1/3) = 18.43^\circ$$

$$\text{Angle } \alpha_{F'} = \alpha_F + 48.43^\circ$$

$$\text{Length } l = \sqrt{3^2 + 1^2} = 3.16$$

$$x' = \cos \alpha_{F'} * l = 2.10$$

$$y' = \sin \alpha_{F'} * l = 2.36$$

## B Kinematic Modeling and Control Theory (Small and Medium Group)

3. The following equation is the kinematic model of a rear wheel drive vehicle as used by Samson.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ \tan \phi / l \\ 0 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2$$

The initial configuration at time  $t_0 = 0$  is ( $x = 20$  cm,  $y = 5$  cm,  $\theta = 60^\circ$ ) and the control inputs are ( $v_1 = 20 \frac{\text{cm}}{\text{s}}$ ,  $\phi = 30^\circ$ ).

What is the new configuration  $x, y, \theta$  of a vehicle with an axle distance  $l$  of 10 cm at time  $t_1 = 0.5$  seconds. [5 marks]

$$\dot{x} = \cos \theta * v_1 = \cos 60^\circ * v_1 = 10 \frac{\text{cm}}{\text{s}}$$

$$\dot{y} = \sin \theta * v_1 = 17.33 \frac{\text{cm}}{\text{s}}$$

$$\dot{\theta} = (\tan \phi) / l * v_1 = 1.15 \text{ Radians} = 1.15 / \pi * 180 = 65 \frac{\text{degree}}{\text{s}}$$

$$x = x + \dot{x} * 0.5 = 25.00 \text{ cm}$$

$$y = y + \dot{y} * 0.5 = 13.67 \text{ cm}$$

$$\theta = \theta + \dot{\theta} * 0.5 = 92.94^\circ$$

## C Fuzzy Logic and Fuzzy Inference (Small and Medium Group)

4. Given are three variables  $x, y, z$ , which take on values in the interval from 0 to 10 and three fuzzy sets Slow, Medium, Fast as defined below.

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QUESTION/ANSWER SHEET

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$$\text{Slow}(x) = 1 - \frac{x}{10}$$

$$\text{Medium}(x) = \begin{cases} \frac{x}{5} & \text{if } x \leq 5 \\ 2 - \frac{x}{5} & \text{else} \end{cases}$$

$$\text{Fast}(x) = \frac{x}{10}$$

Calculate the degree of membership of all three sets for  $x = 6$ . [3 marks]

$$m\text{Slow}(6) = 0.4 \quad m\text{Medium}(6) = 0.8 \quad m\text{Fast}(6) = 0.6$$

5. Given the fuzzy sets as defined in questions 4 and the rules shown below, complete the table to calculate the  $\alpha$ -values for all rules, using Min inferencing.

Rule 1: if  $x$  is slow and  $y$  is slow then  $z$  is slow

Rule 2: if  $x$  is medium and  $y$  is slow then  $z$  is fast

Rule 3: if  $x$  is slow and  $y$  is medium then  $z$  is medium

[5 marks]

x	y	mSlow(x)	mMed(x)	mSlow(y)	mMed(y)	$\alpha_1$	$\alpha_2$	$\alpha_3$
2	6	0.8	0.4	0.4	0.8	0.4	0.4	0.8
4	4	0.6	0.8	0.6	0.8	0.6	0.6	0.6

6. Show the fuzzy subset that is assigned to  $z$  for each of the three rules for  $x = 2, y = 6$ . [4 marks]

$$\text{Rule 1: } z = \begin{cases} 1 - x/10 & \text{if } x \geq 8 \\ 0.2 & \text{else} \end{cases}$$

$$\text{Rule 2: } z = \begin{cases} x/10 & \text{if } x \leq 4 \\ 0.4 & \text{else} \end{cases}$$

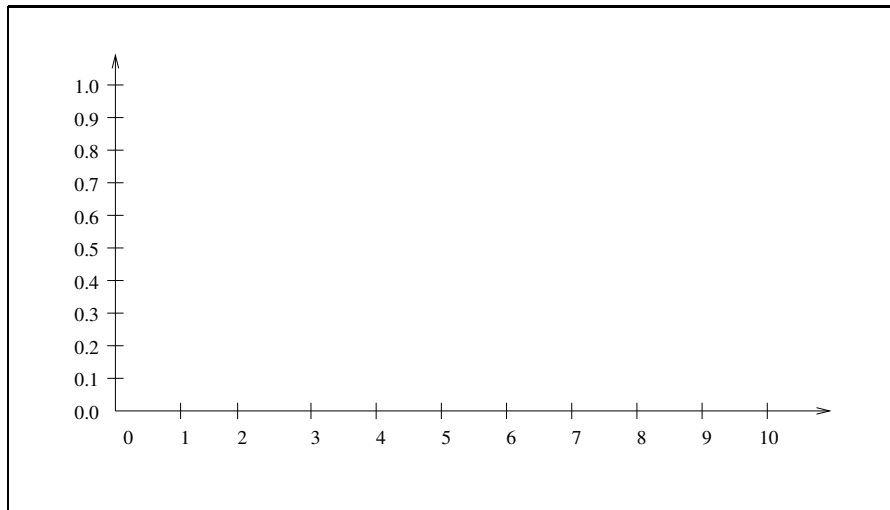
$$\text{Rule 3: } z = \begin{cases} x/5 & \text{if } x \leq 4 \\ 0.8 & \text{if } x > 4 \text{ and } x < 6 \\ 2 - x/5 & \text{else} \end{cases}$$

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7. Using Max composition, show the fuzzy subset that is assigned to  $z$  by composing the results of **all** three rules. [3 marks]



8. Using centroid defuzzification, what is the crisp output value of  $z$ ? [5 marks]

$$c = \frac{0 * 0.4 + 1 * 0.4 + 2 * 0.4 + 3 * 0.6 + 4 * 0.8 + 5 * 0.8 + 6 * 0.8 + 7 * 0.6 + 8 * 0.4 + 9 * 0.2 + 10 * 0.2}{0 + 0.4 + 0.4 + 0.6 + 0.8 + 0.8 + 0.8 + 0.6 + 0.4 + 0.2 + 0.2}$$
$$= \frac{26.2}{5.2} = 5.03$$

## D Reinforcement Learning (Small and Medium Group)

The figure below shows a small environment for a mobile robot. The initial position is at the position ( $B0$ ) and the goal is in the upper right corner ( $A2$ ). The agent can move either vertically or horizontally as indicated by the arrows. Obstacles are marked in grey.

Q-learning uses the following equation to update the Q-values for transitions from state  $i$  to  $j$ .

$$Q(a, i) = R(i) + \alpha(\max_{a'} Q(a', j))$$

where  $Q(a, i)$  is the value of action  $a$  in state  $i$ ,  $\alpha$  is a discount factor,  $R(i)$  is the reward in state  $i$ .

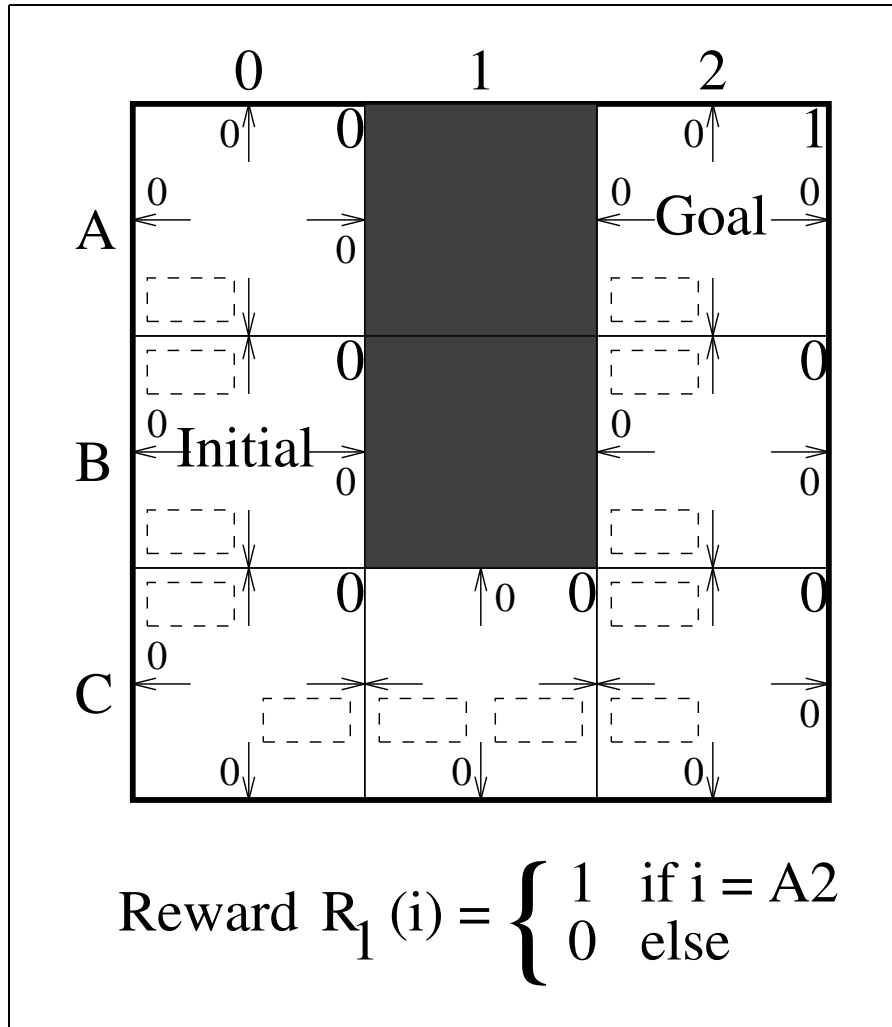
9. Traditional reinforcement learners use a reward function  $R_1$  similar to the one shown in the box below. The reward is 1 if the state is the goal state, 0 otherwise. The values of the reward function for the states are shown in the top right corner. Given this reward function, enter in the dashed boxes the Q values for all actions. In this question the discount factor  $\alpha$  is 0.7, To get you

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started some of the Q values (e.g.,  $Q(\text{up}, A2)=0$ ,  $Q(\text{down}, C1)=0$ ) are already given.

[8 marks]

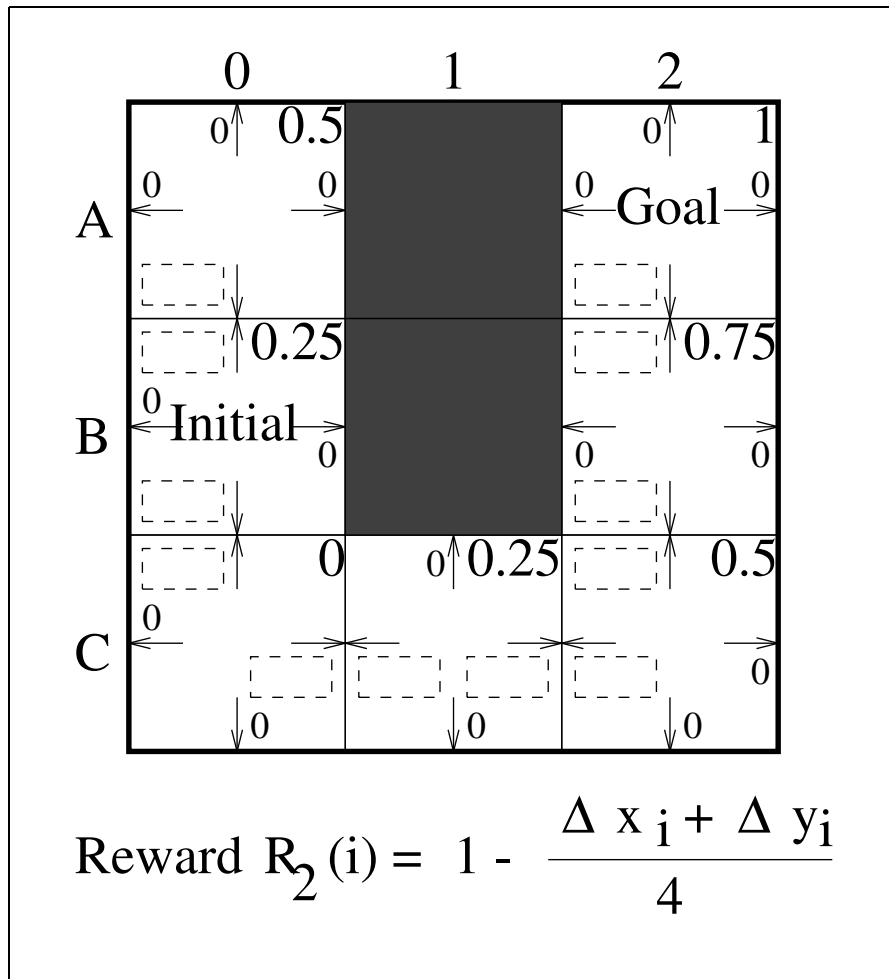


10. Another researcher suggests to use the Manhattan distance as a reward function  $R_2$ .

The Manhattan distance is the sum of vertical ( $\Delta y$ ) and horizontal ( $\Delta x$ ) distances. For example, the distance between the initial state  $B0$  and the goal state  $A2$  is  $2 + 1 = 3$ . To use this function as a reward function it is normalized and inverted as shown in the following figure.

The values of the reward function for the states are shown in the top right corner. Show the Q values in the dashed boxes when using the Manhattan distance as a reward function.

[8 marks]



11. Name one advantage and one disadvantage of a heuristic function as reward function  $R_2$  over a traditional reward function  $R_1$ .

[4 marks]

Advantage= quicker conversion Disadvantage= will not learn the correct function, since it will get stuck following the heuristic function.

## E MC68HC11F1 Embedded System Design (Medium Group only)

The following questions are supposed to test your knowledge of embedded system design in general and MC68HC11F1 assembly language programming in particular.

12. A LED is connected to pin PA6 of the MC68HC11F1 MCU. The LED will