THE UNIVERSITY OF AUCKLAND

EXAMINATION FOR MSc ETC 2000

COMPUTER SCIENCE

Intelligent Active Vision

(Time allowed: TWO hours)

NOTE: Answer questions A,B,C,D,E,F and either question G or question H
Put the answers in the boxes below the questions.
Use of calculators is *permitted*
This is an *open* book exam

SURNAME:

FORENAME(S):

STUDENT ID:

LOGIN:

TOTAL MARKS (OUT OF 100):

Marks:

A	B	C	D	E	F	G or H	Total
15	10	10	20	20	10	15	100

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A Camera Calibration (Small and Medium Groups)

1. A sequence S_1 of rotation and scaling transformations is applied to a **two-dimensional** coordinate system. Given the homogeneous coordinate representation, this sequence can be represented by a 3×3 matrix A_1.

   The coordinates of a point in the new coordinate system $v^* = [kX^*, kY^*, k]^T$ can be obtained from the old coordinates $v = [kX, kY, k]^T$ by the formula $v^* = A_1 v$.

   Show the shape of the matrix A_1. Use $u_1, u_2, \ldots$ for unknown values.

   [5 marks]

$$A_1 = \begin{bmatrix} u_1 & u_2 & 0 \\ u_3 & u_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. The following point pair ($p^* \leftarrow p$) represent the fact that $p^* = A_1 * p$. Does the point pair

$$\begin{pmatrix} -2.73 \\ 4.18 \end{pmatrix} \leftarrow \begin{pmatrix} 3.00 \\ 4.00 \end{pmatrix}$$

   contain sufficient information to determine the transformation matrix A_1 for the sequence S_1?

   If your answer is yes, then explain why another point is not needed. If your answer is no, specify how many additional points are needed and specify a suitable set of points that together with the point (3, 4) will allow to uniquely determine the matrix A_1.

   [10 marks]

There is insufficient information in one point pair. Any other point that is not on the line (0,0) $\rightarrow$ (3,4) will do.

B Kinematic Modeling and Control Theory (Small and Medium Group)

3. The following equation is the kinematic model of a rear wheel drive vehicle as used by Samson.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \tan\phi/l \\ 0 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2$$

The initial configuration at time $t_0 = 0$ is $(x = 50 \text{ cm}, y = 30 \text{ cm}, \theta = 30^o)$ and the control inputs are $(v_1 = 40\frac{cm}{s}, \phi = -15^o)$.

What is the new configuration x, y, θ of a vehicle with an axle distance l of 10 cm at time $t_1 = 0.1$ seconds. [10 marks]

$$\begin{aligned}
\dot{x} &= \cos\theta * v_1 = \cos 30^o * v_1 = 34.64\frac{cm}{s} \\
\dot{y} &= \sin\theta * v_1 = 20\frac{\text{cm}}{\text{s}} \\
\dot{\theta} &= (\tan\phi)/l * v_1 = -1.07 \text{ Radians} = -1.07/\pi * 180 = -61.41\frac{\text{degree}}{\text{s}} \\
x &= x + \dot{x} * 0.1 = 53.46 \text{ cm} \\
y &= y + \dot{y} * 0.1 = 32.00 \text{ cm} \\
\theta &= \theta + \dot{\theta} * 0.1 = 23.86^o
\end{aligned}$$

C Control Theory

4. Balluchi et al. suggest a controller for a Dubin's car which is based on the following control law

$$\omega = \text{sign}(\hat{R}(t))\text{sign}(\sigma)\frac{u(t)}{R_{min}}$$

where σ is:

$$\sigma(\tilde{y}, \tilde{\theta}) = -\frac{\tilde{y}}{R_{min}} - \text{sign}(\tilde{\theta})(1 - \cos(\tilde{\theta}))$$

Assume that the car diverges from a line on a straight line with $\tilde{\theta} = -25^o$. Calculate the distance from the line $\tilde{y}$, when the controller changes the steering angle? Use $R_{min} = 28$cm.

[10 marks]

The controller changes the steering angle when σ is equal to 0. Putting the values for θ into the equation yields:

$$\begin{aligned} 0 &= \frac{\tilde{y}}{R_{min}} + (1 - cos(-25^o)) \\ \tilde{y} &= 0.09 * R_{min} \\ \tilde{y} &= 2.62\text{cm} \end{aligned}$$

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D Fuzzy Logic and Fuzzy Inference (Small and Medium Group)

5. Given are three variables x, y, z, which take on values in the interval from 0 to 10 and three fuzzy sets Small, Medium, Large as defined below.

$$\text{Small}(x) = \begin{cases} 1.0 \text{ if } 0 \leq x < 2 \\ 1.0 - \frac{x-2}{6} \text{ if } 2 \leq x < 8 \\ 0.0 \text{ else} \end{cases}$$

$$\text{Medium}(x) = \begin{cases} 0.0 \text{ if } 0 \leq x < 2 \\ \frac{x-2}{2} \text{ if } 2 \leq x < 4 \\ 1.0 - \frac{x-4}{2} \text{ if } 4 \leq x < 6 \\ 0.0 \text{ else} \end{cases}$$

$$\text{Large}(x) = \frac{x}{10}$$

Calculate the degree of membership of all three sets for $x = 3$. [3 marks]

mSmall(3) = 0.83   mMedium(3) = 0.5   mLarge(3) = 0.3

6. Given the fuzzy sets as defined in questions 5 and the rules shown below, complete the table to calculate the α-values for all rules, using Min inferencing.

Rule 1: if x is small and y is small then z is small
Rule 2: if x is medium and y is medium then z is large
Rule 3: if x is medium and y is small then z is medium

[5 marks]

x	y	mSmall(x)	mMed(x)	mSmall(y)	mMed(y)	α1	α2	α3
3	6	0.83	0.5	0.33	0.0	0.33	0.00	0.33

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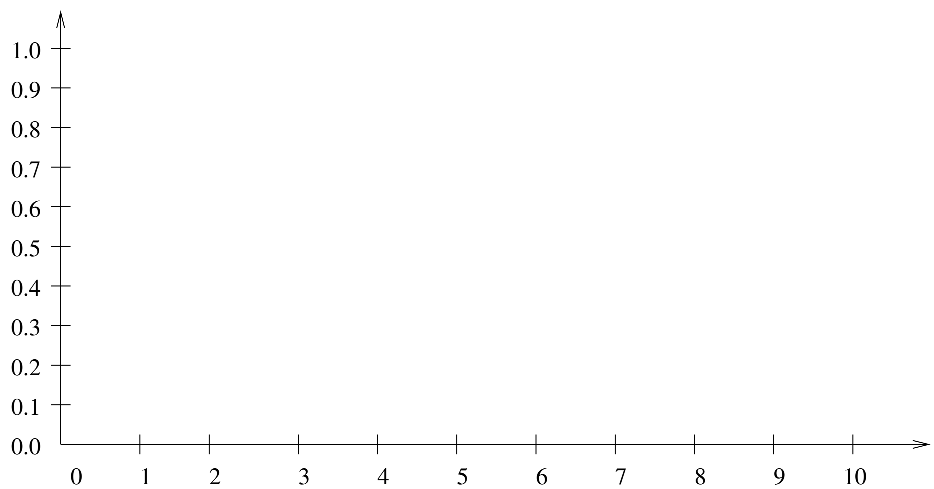
7. Show the fuzzy subset that is assigned to z for each of the three rules for $x = 3, y = 6$. [4 marks]

$$\text{Rule 1:} z = \begin{cases} 0.33 \text{ if } 0 \leq x < 6 \\ 1 - \frac{x-2}{6} \text{ if } 6 \leq x < 8 \\ 0 \text{ if } 8 \leq x < 10 \end{cases}$$

$$\text{Rule 2:} z = 0$$

$$\text{Rule 3:} z = \begin{cases} 0 \text{ if } 0 \leq x < 2 \\ \frac{x-2}{2} \text{ if } 2 \leq x < 2.66 \\ 0.33 \text{ if } 2.66 \leq x < 5.33 \\ 1 - \frac{x-4}{2} \text{ if } 5.33 \leq x < 6 \\ 0 \text{ if } 6 \leq x < 10 \end{cases}$$

8. Using Max composition, show the fuzzy subset that is assigned to z by composing the results of **all** three rules. [3 marks]



9. Using centroid defuzzification, what is the crisp output value of z? [5 marks]

$$\begin{aligned} c &= \frac{0*0.33 + 1*0.33 + 2*0.33 + 3*0.33 + 4*0.33 + 5*0.33 + 6*0.33 + 7*0.17 + 8*0.0 + 9*0.0 + 10*0.0}{0.33 + 0.33 + 0.33 + 0.33 + 0.33 + 0.33 + 0.33 + 0.17 + 0.00 + 0 + 0} \\ &= \frac{8.12}{2.48} = 3.27 \end{aligned}$$

E Path Planning

10. A mobile robot can move in a four-connected environment, that is it can move up, down, left or right. Given the following environment, show the value of the potential field for each square.

    The goal point exerts a positive potential using the function.

    $$F_{\text{Goal}} = 10 - d \ \text{ if } 0 \leq d < 10; \ 0 \ \mathit{else}$$

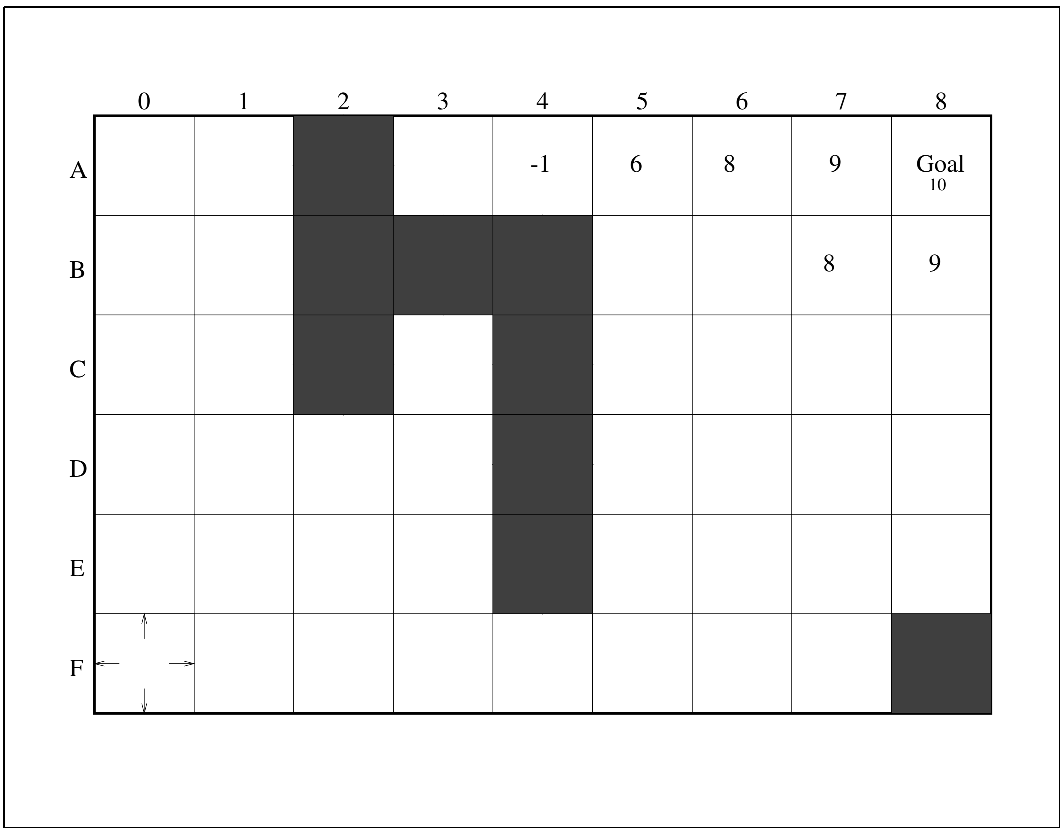
    where d is the Manhatten distance between the square and the goal.

    Each part of an obstacle exerts a repelling potential proportional to the square of the distance, as shown in the following equation.

    $$F_{\text{Obst}} = -(5 - d^2) \ \text{ if } 0 \leq d \leq 2; \ 0 \ \mathit{else}$$

    Show the potential field value for all squares in the following field. Some of the values are already shown to get you started.

    [15 marks]



11. Identify at least one local minima in the potential field.

[2 marks]

There is a local minimum at square: A0

12. Identify at least one initial position, where a hill climbing algorithm will be unable to reach the goal position. Hill climbing is a standard search technique and often used in conjunction with potential fields. A hill climbing algorithm follows the steepest gradient in an attempt to reach higher (better) locations. So for each position, the robot will move to the neighbor with the highest potential field value.

[3 marks]

A path from square B0 will not reach the goal

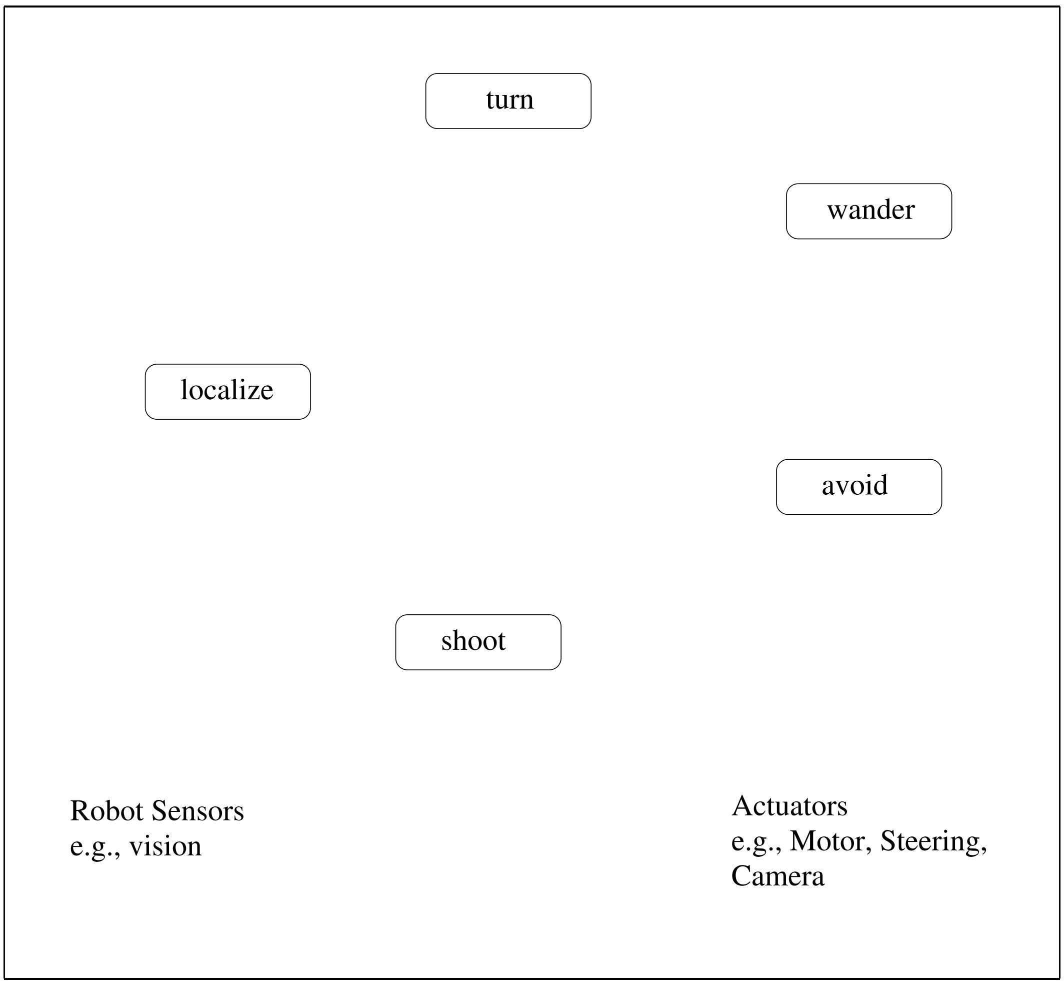
F Subsumption Architecture

13. You want to design an intelligent soccer playing robot based . From a previous design, you can use the following behaviors:

    - `turn` if the robot sees the ball, then turn towards it.
    - `wander` move around at random
    - `avoid` an obstacle if it is in your path
    - `shoot` if the ball is in front of you, then shoot
    - `localize` determine whether we are facing the opponents goal or not.

    Design a subsumption architecture using the behaviors mentioned above for a striker. To coordinate your behaviors, you can use *inihibition* or *suppression* links.

    [10 marks]



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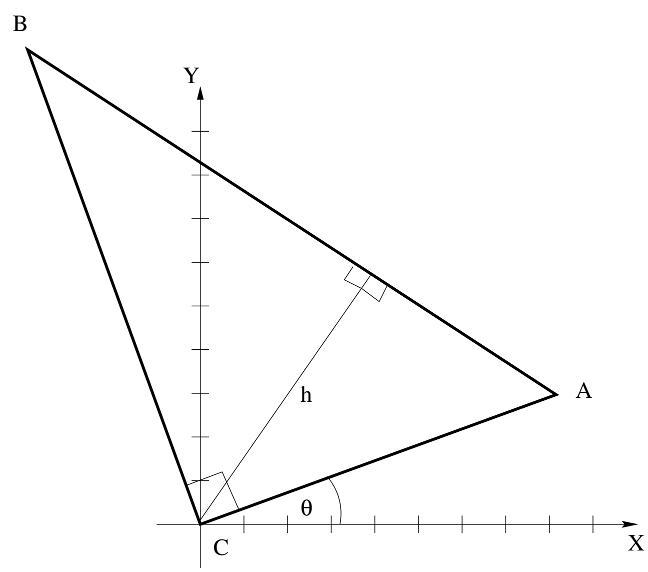
G Hough Transform

14. A generalized hough transform is to be used to determine the orientation of a right angle triangle ABC. The position of corner `C` (right angle) is known and a local coordinate system with corner `C` at the origin is established. The orientation of the triangle ABC is determined by θ, the angle between the side AC and the X-axis.

    An edge detection operator is used to determine the edge pixels of the shape. All these edge pixels are then used in the Hough transform.

    An example is shown in the figure below.



What is the dimension of the Hough space in this example? In other words, what datastrucuture would you use for the Hough space? [2 marks]

I would use a one-dimensional array of integers

15. What is the maximum number of orientations that are supported by an edge pixel at a position.

[3 marks]

A pixel with d > 3 supports at most four possible orientations.

16. Assume that the sides of the triangle are

$$AC = 6 \text{ cm}$$
$$BC = 8 \text{ cm}$$
$$AB = 10 \text{ cm}$$

What possible orientations of the triangle are supported by an edge pixel at position $D = (2, 4)$.

[10 marks]

$\theta_1 = 45^o$
$\theta_2 = -45^o$

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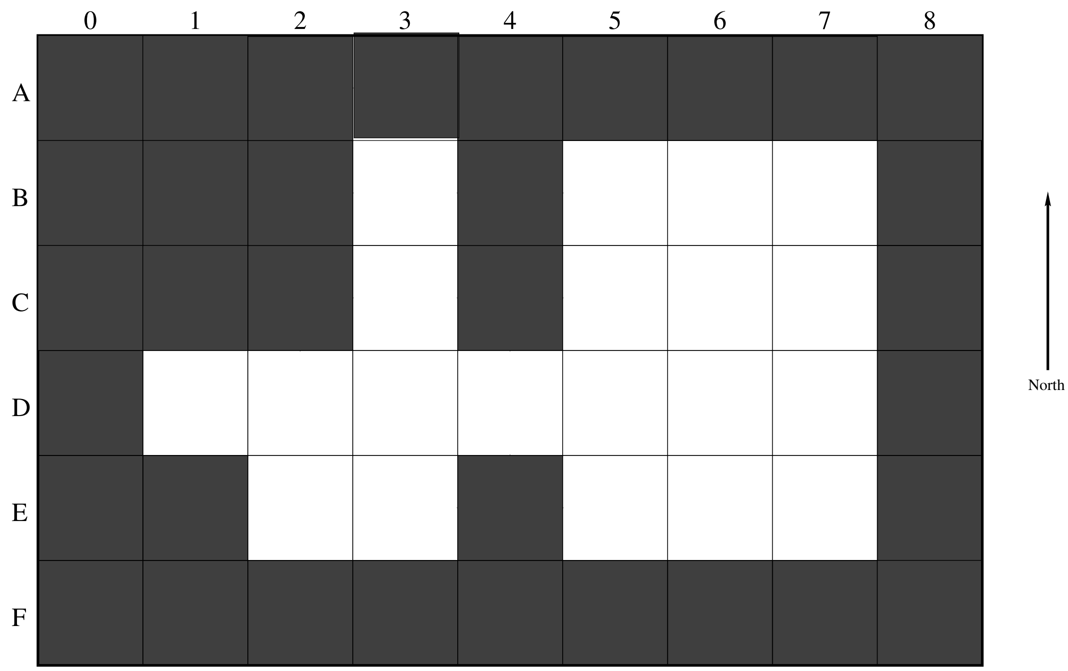
H Localization

17. In the following question, assume that a robot has sensors for its orientation and that it can detect obstacles in four directions. For example, the sensor input for location B3 is `North=Obstacle, East=Obstacle, South=Free, West=Obstacle`.

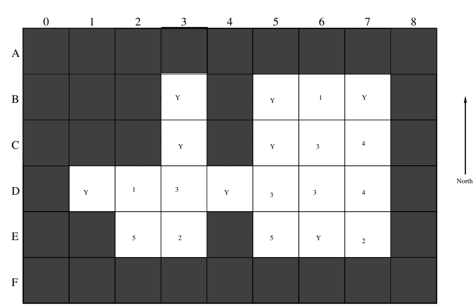
    The robot is in the environment shown below and starts at a random initial position.



Identify all positions, where the robot can localize itself correctly on the map using just its current sensor input.

[5 marks]

The robot can localize itself in positions:



18. Using a history, that is remembering sensor readings in previous states and the transitions between states is often used to localize a robot in states that are not unique. Given the environment above and assuming that the robot does not move in a cycle, how many states need to be contained in the history so that the robot can localize itself completely? Briefly justify your answer.

[10 marks]

The length of the history must be at least 2 states, because