
THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2001
Campus: Tamaki

COMPUTER SCIENCE

Intelligent Active Vision

(Time allowed: TWO hours)

NOTE: Attempt questions A, B, C, D, E and *either* question F or G.
 This is an *open* book examination.
 Use of calculators is *permitted*.
 Show your working to receive full marks.

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FORENAME(S): _____

STUDENT ID: _____

A	B	C	D	E	F or G	Total
10	20	10	20	20	20	100

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Section A: Kinematic Model of Car-Like Mobile Robots (All Groups)

The following equation is the kinematic model of a rear wheel drive vehicle as used by Samson.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \\ \tan \phi / l \\ 0 \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_2$$

1. The state of the car at time $t_1 = 0.01s$ is $x = 10.00$ cm, $y = 10.00$ cm, $\theta = 30.00^\circ$. The car has an axle distance l of 10 cm.

What was the previous position of the car at time $t_0 = 0.00$, assuming that the velocity $v_1 = 10$ cm/s and the steering angle $\phi = -25^\circ$.

[5 marks]

$$\begin{aligned} \dot{\theta} &= \tan(\phi)/l * v_1 = -0.4 \frac{\text{radians}}{\text{s}} = -26.72 \frac{\text{degree}}{\text{s}} \\ \theta_0 &= \theta_1 - \dot{\theta} * 0.01s = 30.02 \\ \dot{x} &= \cos(\theta_0) * v_1 = 8.65 \\ \dot{y} &= \sin(\theta_0) * v_1 = 5.00 \\ x_0 &= x_1 - \dot{x} * 0.01 = 9.91 \\ y_0 &= y_1 - \dot{y} * 0.01 = 9.94 \end{aligned}$$

2. The state of the car at time $t_0 = 0s$ is $x = 20.00$ cm, $y = 40.00$ cm, $\theta = 15.00^\circ$. The car has an axle distance l of 10 cm.

At time $t_1 = 0.01s$, the state of the car is $x = 20.48$ cm, $y = 40.13$ cm, $\theta = 16.65^\circ$.

Compute an approximation of the control inputs v_1 and ϕ .

[5 marks]

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$$\begin{aligned} \dot{x} &= 0.48 \text{ cm}/0.01 \text{ s} = 48 \frac{\text{cm}}{\text{s}} \\ \dot{y} &= 0.13 \text{ cm}/0.01 \text{ s} = 13 \frac{\text{cm}}{\text{s}} \\ \dot{\theta} &= 1.65/0.01 = 165 \frac{\text{degree}}{\text{s}} = 2.87 \frac{\text{radians}}{\text{s}} \\ v_1 &= 48 / \cos \theta = 13 / \sin \theta = 50 \text{ cm/s} \\ \phi &= \text{atan}\left(\frac{\dot{\theta} l}{v_1}\right) = 30^\circ \end{aligned}$$

Section B: Fuzzy Logic and Fuzzy Inference (All Groups)

3. Given are three variables x, y, z , which take on values in the interval from 0 to 10 and three fuzzy sets Slow, Medium, Fast as defined below.

$$\text{Slow}(x) = \begin{cases} 1.0 & \text{if } 0 \leq x < 2 \\ 1.0 - \frac{x-2}{4} & \text{if } 2 \leq x < 6 \\ 0.0 & \text{else} \end{cases}$$

$$\text{Medium}(x) = \begin{cases} 0.0 & \text{if } 0 \leq x < 1 \\ \frac{x-1}{2} & \text{if } 1 \leq x < 3 \\ 1.0 - \frac{x-3}{2} & \text{if } 3 \leq x < 5 \\ 0.0 & \text{else} \end{cases}$$

$$\text{Fast}(x) = \frac{x}{10}$$

Calculate the degree of membership of all three sets for $x = 3$.

[3 marks]

$$m\text{Slow}(3) = 0.75 \quad m\text{Medium}(3) = 1 \quad m\text{Fast}(3) = 0.3$$

4. Given the fuzzy sets as defined in questions 3 and the rules shown below, complete the table to calculate the α -values for all rules, using Min inferencing.

- Rule 1: if x is slow and y is fast then z is slow
 Rule 2: if x is medium and y is medium then z is fast
 Rule 3: if x is medium and y is fast then z is medium

[5 marks]

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x	y	mSlow(x)	mMed(x)	mMed(y)	mFast(y)	α_1	α_2	α_3
3	4	0.75	1.0	0.5	0.4	0.4	0.5	0.4

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5. Show the fuzzy subset that is assigned to z for each of the three rules for $x = 3, y = 4$.

[4 marks]

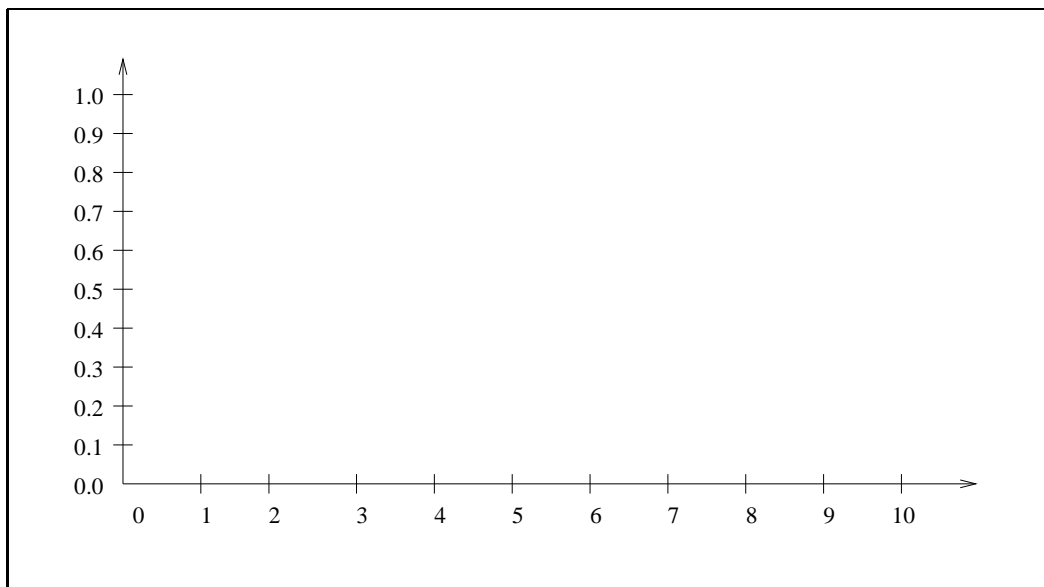
$$\text{Rule 1: } z = \begin{cases} 0.4 & \text{if } 0 \leq x < 4.4 \\ 1 - \frac{x-2}{4} & \text{if } 4.4 \leq x < 6 \\ 0 & \text{else} \end{cases}$$

$$\text{Rule 2: } z = \begin{cases} \frac{x}{10} & \text{if } 0 \leq x < 5 \\ 0.5 & \text{else} \end{cases}$$

$$\text{Rule 3: } z = \begin{cases} 0.0 & \text{if } 0 \leq x < 1 \\ \frac{x-1}{2} & \text{if } 1 \leq x < 1.8 \\ 0.4 & \text{if } 1.8 \leq x < 4.2 \\ 1 - \frac{x-3}{2} & \text{if } 4.2 \leq x < 5 \\ 0 & \text{else} \end{cases}$$

6. Using Max composition, show the fuzzy subset that is assigned to z by composing the results of **all** three rules.

[3 marks]



7. Using centroid defuzzification, what is the crisp output value of z ?

[5 marks]

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$$c = \frac{0 * 0.4 + 1 * 0.4 + 2 * 0.5 + 3 * 0.5 + 4 * 0.5 + 5 * 0.25 + 6 * 0 + 7 * 0 + 8 * 0 + 9 * 0 + 10 * 0}{0.4 + 0.4 + 0.5 + 0.5 + 0.5 + 0.25 + \dots}$$
$$= \frac{6.15}{2.55} = 2.41$$

Section C: Subsumption Architecture (All Groups)

8. You want to design an intelligent soccer playing robot based on a subsumption architecture.

From a previous design, you can use the following behaviors:

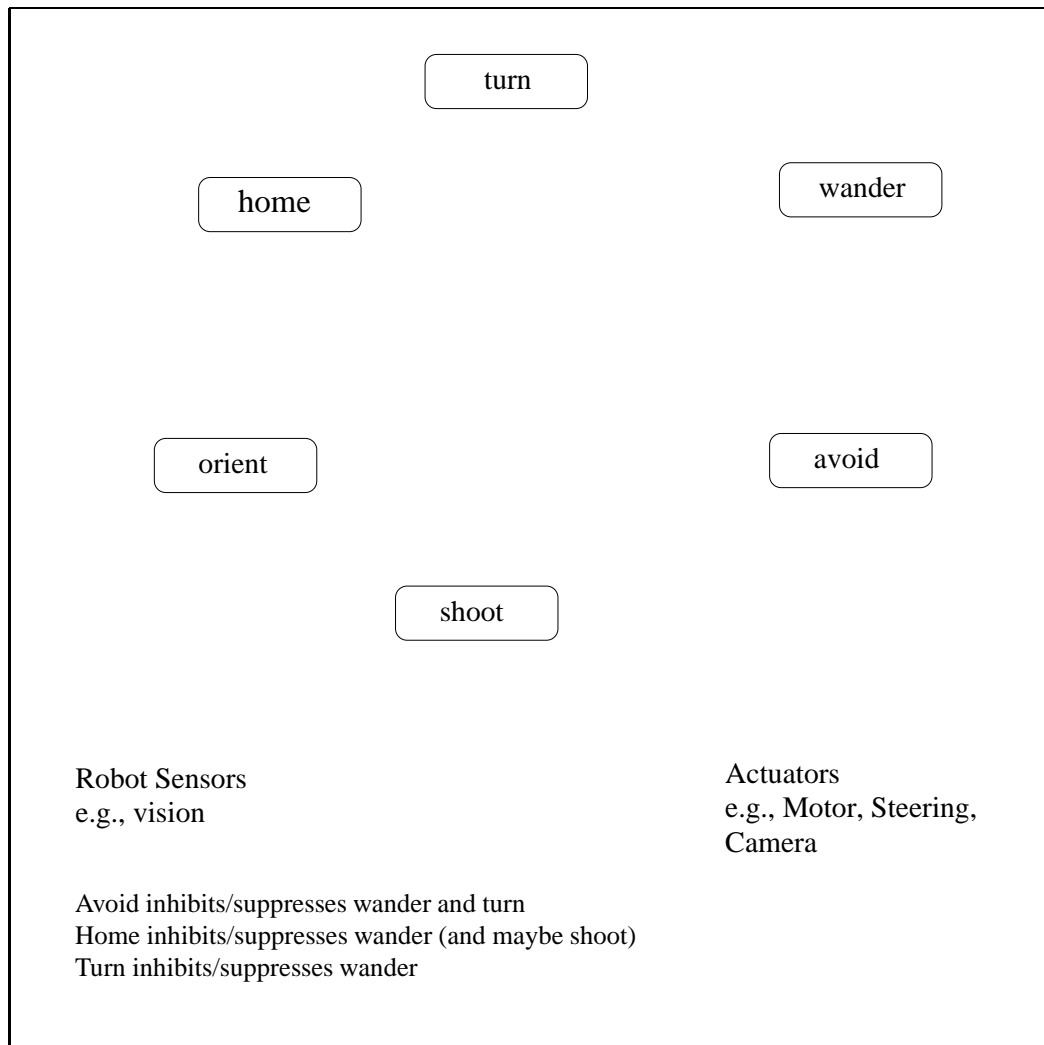
- `turn` if the robot sees the ball, then turn towards it.
- `wander` move around at random
- `avoid` an obstacle if it is in your path by turning randomly
- `shoot` if the ball is in front of you, then shoot
- `home` move towards the home goal, if we can see it
- `orient` orient the robot to face the opponents goal when inside the home goal box

Design a subsumption architecture using the behaviors mentioned above for a `goal_keeper`. To coordinate your behaviors, you can use *inhibition* or *suppression* links.

[10 marks]

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Section D: Path Planning (All Groups)

The following question investigates trade-offs in various path planning algorithms.

9. Quadtree decomposition is a popular method for approximate cell decomposition that breaks the space into quaters recursively.

Show the quadtree decomposition of the following example domain with a maximum resolution of 1 square.

Indicate the label for each region of the quadtree (mixed, free, obstacle).

[7 marks]

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	0	1	2	3	4	5	6	7
A			I					G
B								
C								
D								
E								
F								
G								
H								

Number of regions in the quadtree decomposition: 49

10. Assume that a robot can move only up, down, left, or right. The robot is currently at position A2 and the goal is located at position A7. Indicate the path that the robot would take using the quadtree cell decomposition path planning algorithm.

[5 marks]

Surname: _____

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Path: A2, A/B2/3, C3, D4, E/F2/3,F4,F5,E5,D5,C/D6/7,B7,A7

11. Potential fields are another popular class of path planning algorithms. In this example, the potential field is based on the Manhattan distance between the current position and the goal.

The goal point exerts a positive potential using the function.

$$F_{\text{Goal}} = 10 - d \text{ if } 0 \leq d < 10; 0 \text{ else}$$

where d is the Manhattan distance between the square and the goal.

Each part of an obstacle exerts a repelling potential proportional to the square of the distance, as shown in the following equation.

$$F_{\text{Obst}} = -(5 - d^2) \text{ if } 0 \leq d \leq 2; 0 \text{ else}$$

Show the potential field value for all squares in the following field. Some of the values are already shown to get you started.

[8 marks]

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	0	1	2	3	4	5	6	7
A	3	4	4					G
B	1	2	2					
C	-5							
D								
E								

Section E: Search (All Groups)

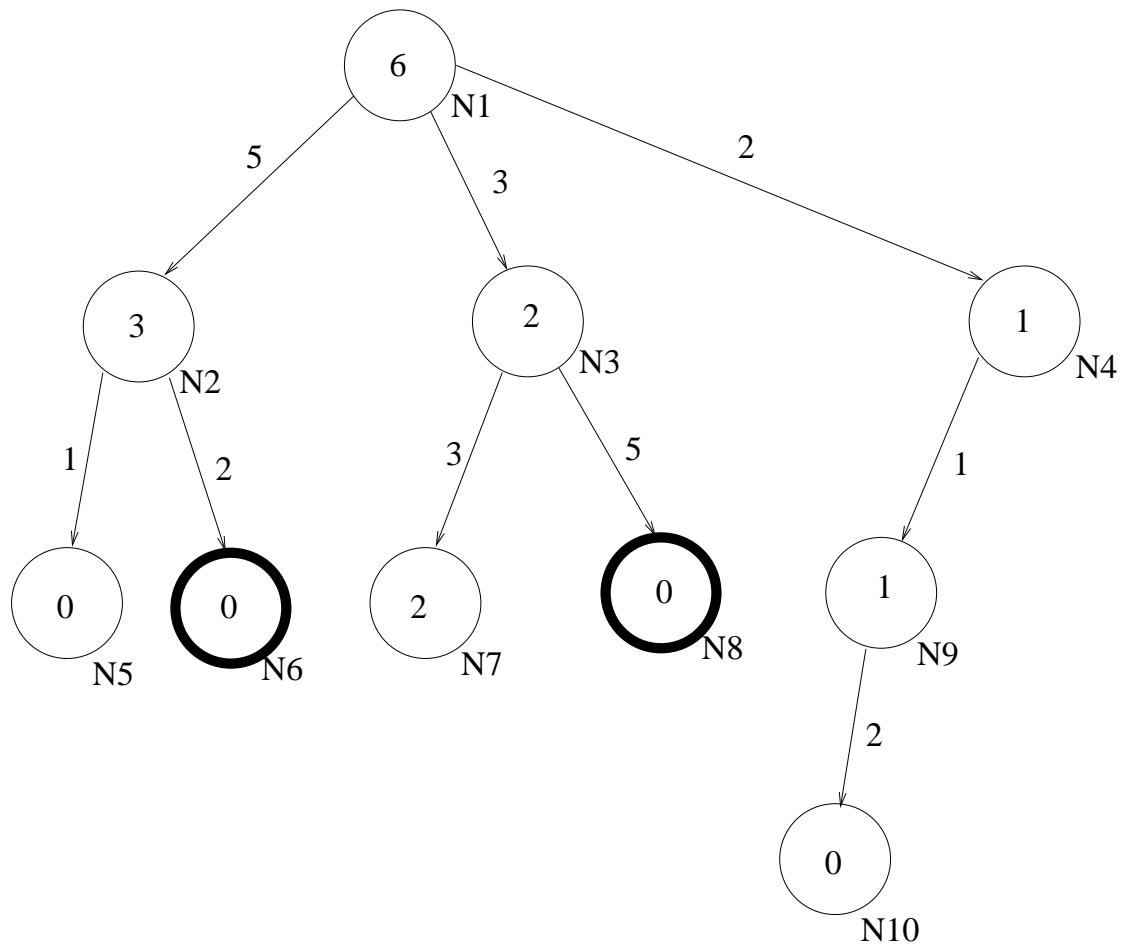
This question investigates different search methods in Artificial Intelligence, in particular A^* and IDA^* search.

The search tree below corresponds to a navigation problem. The numbers along the edges correspond to the actual cost g of traversing this link, numbers inside the node indicate the expected (heuristic) cost from the node to the goal node.

The goal node of the search are nodes $N6$ and $N8$.

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12. Show the order in which the A^* algorithm expands the nodes in the example tree.

[7 marks]

One sample expansion: N1, N4, N9, N3, N8

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13. Is the heuristic function used an *admissible* heuristic function, i.e., it is an underestimate of the true cost to reach the goal? Justify your answer briefly.

[3 marks]

No. The heuristic value in Node N2 is *not* an underestimate.

14. Describe the effect that choosing a non-admissible heuristic has on the quality of the solution.

[2 marks]

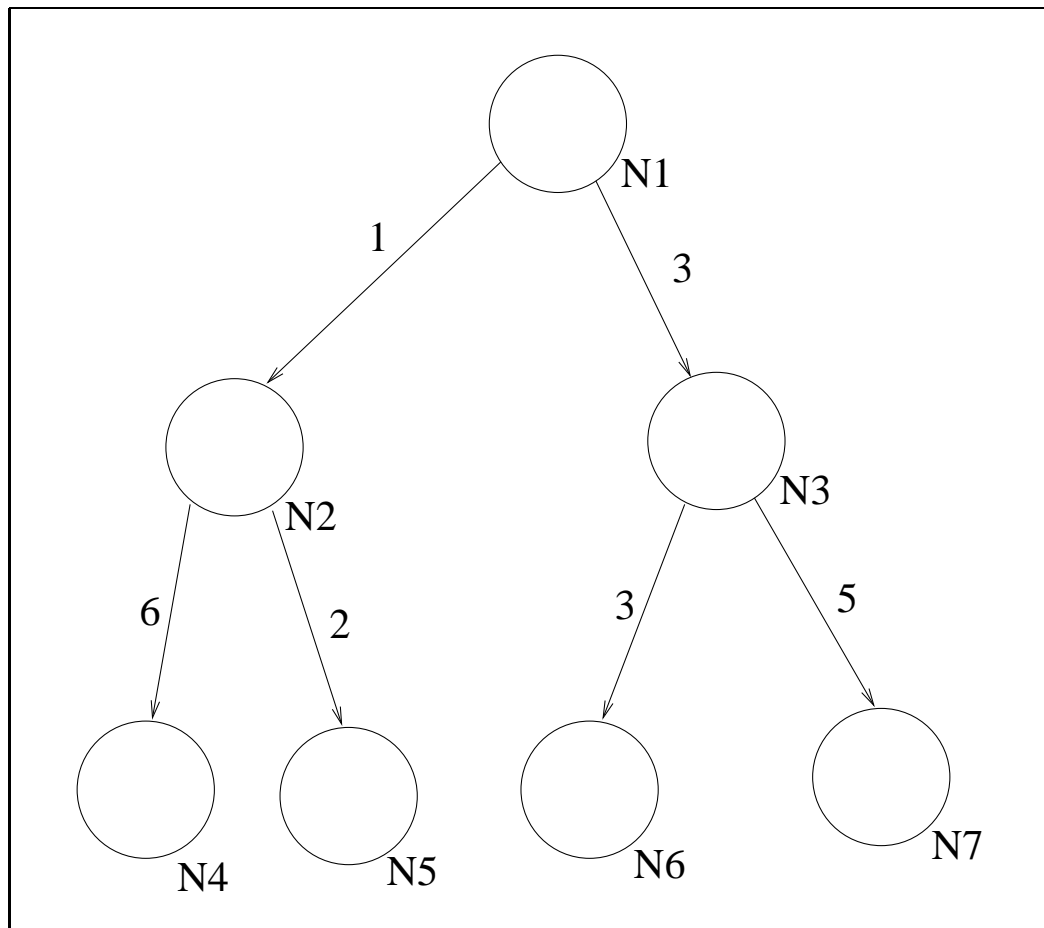
It leads to a non-optimal solution as is shown in this example.

15. The figure in the answerbox below contains a small search space. The cost function for each node is indicated beside the arc, but the values of the heuristic function for some of the nodes are missing. Show the values of the heuristic function for all nodes, such that the search space is traversed in depth-first order (i.e., the nodes are expanded in the order N1, N2, N4, N5, N3, N6, N7).

[8 marks]

Surname: _____

Forename(s): _____



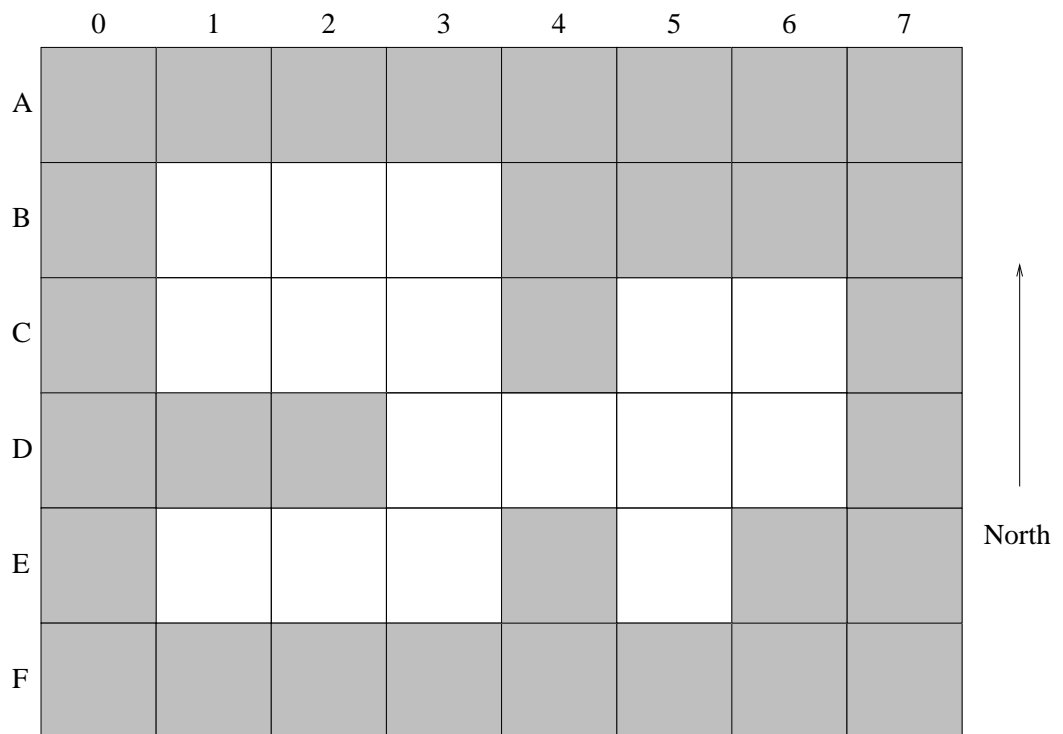
Section F: Localization (Local Vision Groups)

16. In the following question, assume that a robot can detect obstacles in four directions. For example, the sensor input for location B3 is North=Obstacle, East=Obstacle, South=Free, West=Free.

The robot is in the environment shown below and starts at a random initial position.

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Identify all positions, where the robot can localize itself correctly on the map using just its current sensor input.

[10 marks]

The robot can localize itself in positions: $B2, C1, C2, C3, D3, D5, E1, E5$.

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17. Using a history, that is remembering sensor readings in previous states and the transitions between states is often used to localize a robot in states that are not unique. Given the environment above and assuming that the robot does not move in a cycle, how many states need to be contained in the history so that the robot can localize itself completely? Briefly justify your answer.

[10 marks]

The length of the history must be at least 1 state, because for any state that is not localizable, any possible next state is either fully localizable or leads to a difference sequence of states, which is unique in this environment.

Section G: Computer Vision for Robotics (Global Vision Groups)

The focus of this section is the problem of finding position, orientation, and identity of mobile robots.

18. Assume a robot with a triangular shape. Side a and b of the robot are 15cm long. Side c is 10cm long.

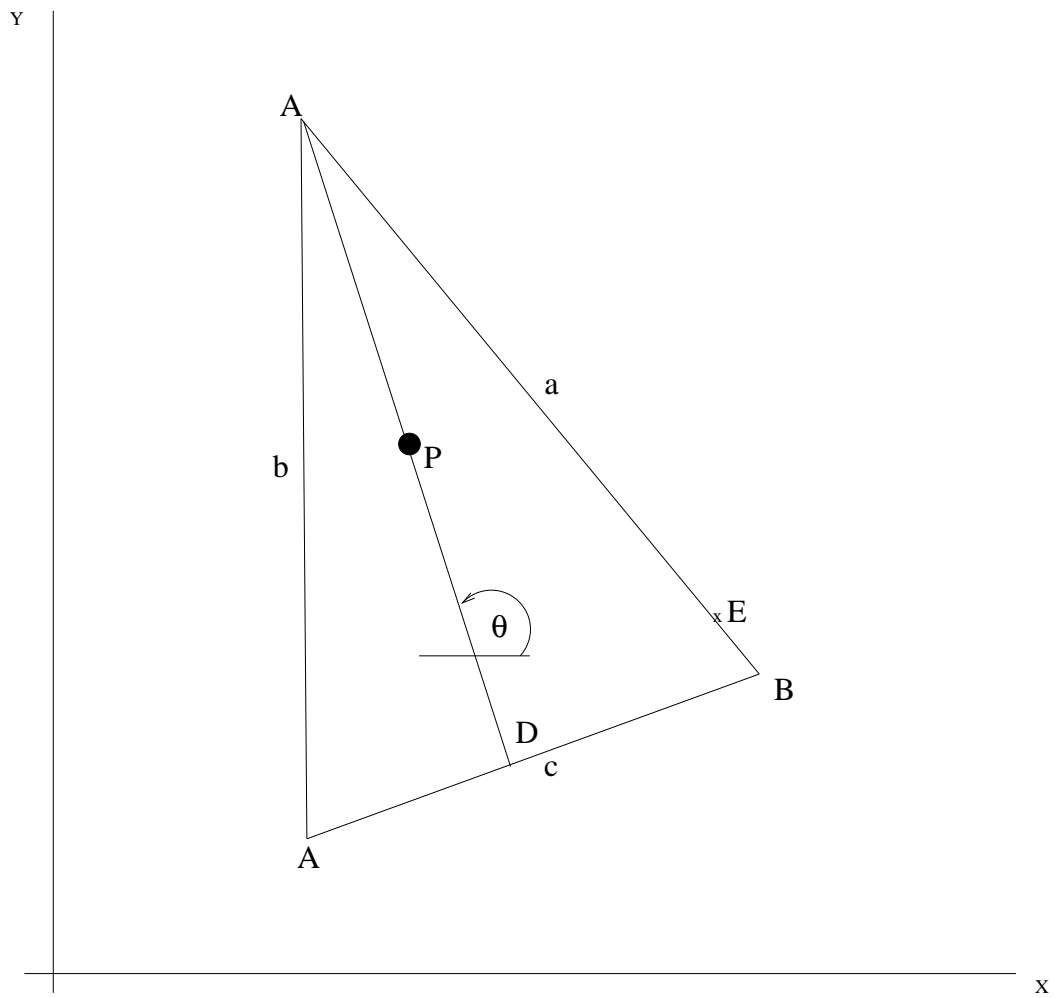
The position of the robot is determined through a coloured marker in the center of the robot, that is half way between points C and D .

The center of the robot is at location $P = (x = 8 \text{ cm}, y = 10 \text{ cm})$. An edge pixel e is detected at location $E = (x = 13 \text{ cm}, y = 6 \text{ cm})$.

One possible arrangement is shown in the figure below.

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The orientation θ of the robot is the angle between the X axis and the line *overline*DA.

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19. Assume that you are using a generalized hough transform to detect the orientation of the robot. Furthermore, you want to detect the orientation of the robot with a resolution of 5 degrees. Indicate the dimensions and the number of entries in the hough space.

[10 marks]

20. Show the maximum number of possible orientations that are supported by a single edge pixel.

[10 marks]

A single edge pixel supports at most 6 orientations

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Additional work pages

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Additional work pages