
UNIVERSITY OF MANITOBA

Final Examination

Winter 2004

COMPUTER SCIENCE

Machine Learning

Paper No.: 234
Examiners: Jacky Baltes
Date: 14 April 2004
Time: 18:00
Room: University College, Great Hall

(Time allowed: 180 Minutes)

NOTE: Attempt all questions.
This is a *closed* book examination.
Use of calculators is *permitted*.
Show your work to receive full marks.

SURNAME:

FORENAME(S):

STUDENT ID:

A	B	C	D	E	Total
20	20	20	20	20	100

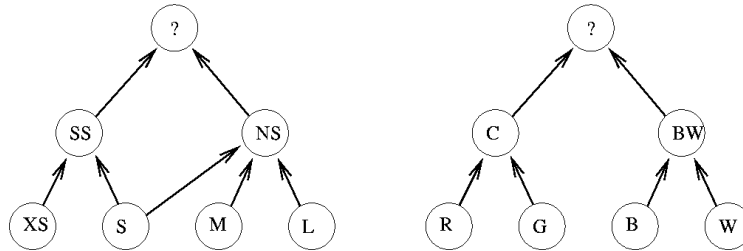
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Section A: Candidate Elimination

1. Given the hypothesis space H below, the candidate elimination algorithm is trained on a sequence of instances (Training data D).



Given the generalization hierarchy H in question 1, show a trace (i.e., S-set and G-set) of the candidate elimination algorithm for the following five instances. If it is impossible to trace the execution of the candidate elimination algorithm based on the information above, then say so in your answer and explain why.

[5 marks]

```

<XS, R> -
S-Set: 0
G-Set: <NS, ?>, <?, G>, <?, BW>
<M, G> +
S-Set: <M, G>
G-Set: <NS, ?>, <?, G>
<L, G> +
S-Set: <NS, G>
G-Set: <NS, ?>, <?, G>
<L, B> -
S-Set: <NS, G>
G-Set: <?, G>
<S, G> -
S-Set: 0
G-Set: 0
    
```

2. Given the generalization hierarchy H in question 1, assume that after a training sequence D , the candidate elimination algorithm returns the following version space V :

S-set : <M, BW>
 G-set : <NS, ?>, <?, BW>

How many concepts consistent with version space V classify the instance <L, W> as positive and how many classify it as negative? If it is impossible to determine the classification from the information above, then say so in your answer and explain why it is impossible.

[5 marks]

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<L, W> is classified as positive by 3 concepts and negative by 2 concepts.
 <M, BW>-, <M, ?>-
 <NS, BW>+, <NS, ?>+, <?, BW>+

3. Given the generalization hierarchy H in question 1, the candidate elimination algorithm is trained on the training sequence D' and returns the version space V' :

S-set : <L, G>
 G-set : <NS, ?>

What is the minimum number of instances in the training sequence D' ? Show a minimal training sequence D' which results in the version space V' . If no such sequence exists, then say so in your answer and explain why.

[5 marks]

The minimum number of instances in D to create V' is: 2
 Instance Classification
 <L, G> +
 <XS, G> -

4. What is the maximum number of semantically different concepts that can be expressed given the generalization hierarchy H in question 1? If it is impossible to determine the maximum number of semantically different concepts in H , then say so in your answer and explain why.

[5 marks]

The maximum number of semantically different concepts in H is: $1 + 7 * 7 = 50$.

Section B: Decision Trees

The information gain $\text{Gain}(S, A)$ of an attribute A for a sample set S is defined as

$$\text{Gain}(S, A) = \text{Entropy}(S) - \sum_{v \in \text{Values}(A)} \frac{\|S_v\|}{\|S\|} \text{Entropy}(S_v)$$

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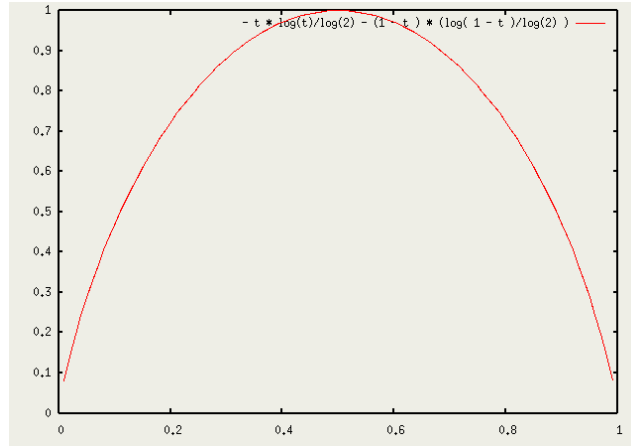


Figure 1: Graph of the Entropy Function

A graph of the entropy function is shown in Fig. 1 below. You can use this graph when answering the following questions.

5. Assume a domain with three attributes A, B, and C. Each attribute has two possible values T and F. Given below is a set of instances.

A	B	C	Target
T	T	T	Yes
T	T	F	No
T	F	T	Yes
F	T	T	Yes
F	T	F	No
F	F	F	Yes

Calculate the information gain ($\text{Gain}(S, ?)$) for the attributes A, B, and C. Which attribute would be selected by the standard ID3 algorithm? If it is impossible to calculate the information gain from the given information, then specify so in your answer and explain why.

[10 marks]

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$$\text{Gain}(S, A) = 0$$

$$\text{Gain}(S, B) =$$

$$\text{Gain}(S, C) =$$

ID3 would select the attribute C

$$\text{ent}(4,2)=0.9182958$$

$$\text{IG}(A)=\text{ent}(4,2) - 3/6 * \text{ent}(2,1) - 3/6 * \text{ent}(2,1) = 0$$

$$\text{IG}(B)=\text{ent}(4,2) - 4/6 * \text{ent}(2,2) - 2/6 * \text{ent}(2,0) = 0.2516292$$

$$\text{IG}(C)=\text{ent}(4,2) - 3/6 * \text{ent}(3,0) - 3/6 * \text{ent}(1,2) = 0.4591479$$

6. You are given the set of instances below. Furthermore, you are told that the ID3 algorithm selected attribute A. Is this sufficient information to *uniquely* (i.e., only one possible solution) determine the missing assignments (T or F) for attribute B? If so, show the assignments. If not, then say so in your answer and explain why.

[10 marks]

A	B	C	Target
T	T	Yes	
T	F	No	
T	T	Yes	
F	T	Yes	
F	F	No	
F	F	Yes	

Given this assignment, it is impossible to get an information gain that is lower than the one for attribute A, since the maximum entropy is will be achieved for F,T. You can also convince yourself of this by calculating the information gain for all four possibilities.

Section C: Neural Nets

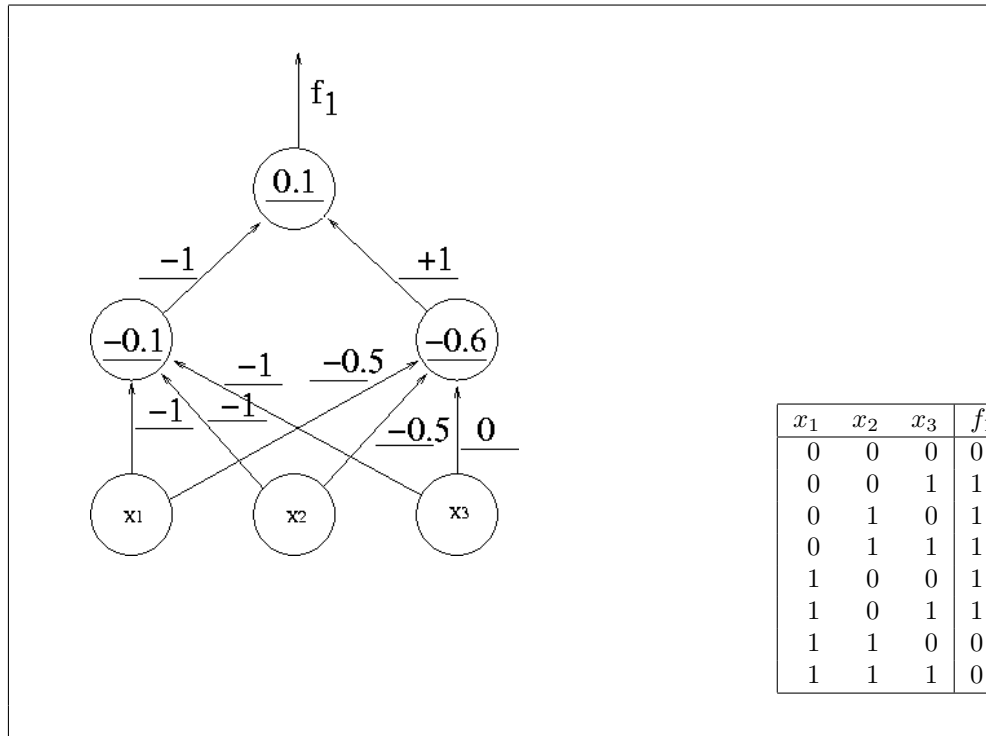
7. Given below is an artificial neural network (ANN) with three input nodes (X_1, X_2, X_3), two hidden nodes, and one output node. The network uses simple threshold nodes (i.e., the node will output 1.0 if the sum of the weighted inputs is greater than the threshold, 0 otherwise).

Show a set of weights and thresholds for all nodes that implement the function f_1 . If it is impossible to represent the function f_1 with the given neural network, then state this in your answer and explain why this is impossible.

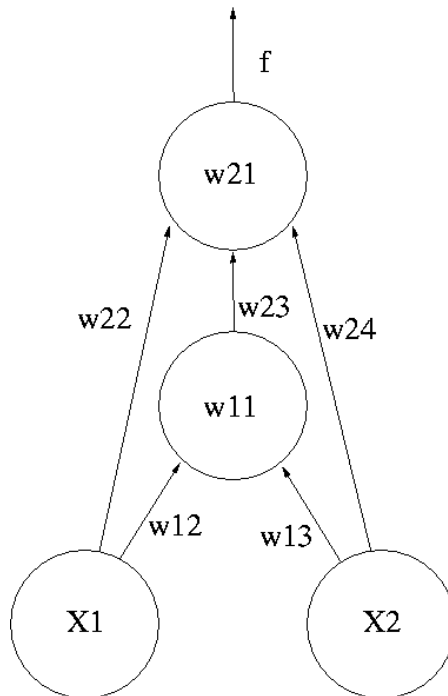
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8. Given below is a figure of an artificial neural net (ANN) with four nodes.



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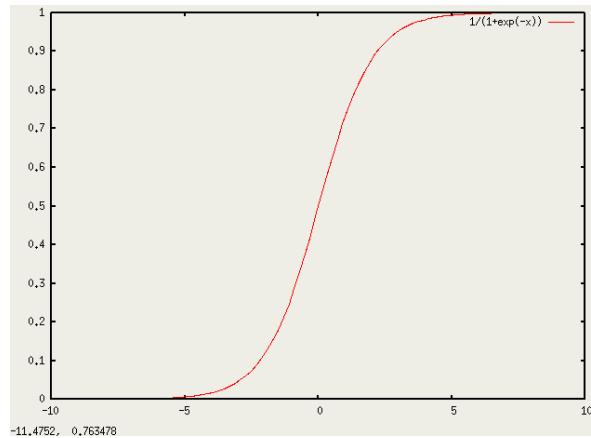
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The ANN uses nodes with the sigmoid activation function. The sigmoid activation function is shown below.

$$\text{activation } a = \sum_{i=0}^n w_i x_i$$

$$\text{output } y = \sigma(a) = \frac{1}{1 + e^{-a}}$$



The thresholds of the nodes are given by w_{11} and w_{21} . You can think of the threshold as being connected to a constant input of -1. The weights of the edges are given as $w_{12}, w_{13}, w_{22}, w_{23}$, and w_{24} respectively.

Given the weights, $w_{11} \dots w_{24}$, the ANN computes the function $f(x_1, x_2)$. The function $f'(x_1, x_2)$ is the “inverse” of $f(x_1, x_2)$. That is $f'(x_1, x_2) = 1 - f(x_1, x_2)$.

Your goal is to modify the weights $w_{11} \dots w_{24}$ in such a way, that the same network computes the function $f'(x_1, x_2) = 1 - f(x_1, x_2)$. Show the new set of weights $w'_{11} \dots w'_{24}$ so that the neural network calculates $f'(x_1, x_2)$ in the answer box below. If it is impossible to calculate the new weights or if no such set of weights exists, then say so in your answer and explain why.

[10 marks]

$$\begin{aligned} w'_{11} &= w_{11} & w'_{21} &= -w_{21} \\ w'_{12} &= w_{12} & w'_{22} &= -w_{22} \\ w'_{13} &= w_{13} & w'_{23} &= -w_{23} \\ & & w'_{24} &= -w_{24} \end{aligned}$$

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Section D: Bayesian Learning

9. Given the following data set, what is the naive Bayesian classification of the new instance $\langle L, \text{white} \rangle$. Show your work for full marks.

Size	Color	Target
XS	green	Yes
L	green	No
XS	white	Yes
M	black	No
XL	green	Yes
XS	white	No
L	black	No
M	green	Yes

Table 1: Training set for the Size, Color domain

[5 marks]

$$\begin{aligned}
 P(\text{Yes}) &= 4/8 = 0.5 \\
 P(\text{No}) &= 4/8 = 0.5 \\
 P(L|\text{Yes}) &= 0/2 = 0 \\
 P(L|\text{No}) &= 2/4 = 0.5 \\
 P(\text{white}|\text{Yes}) &= 1/4 = 0.25 \\
 P(\text{white}|\text{No}) &= 1/4 = 0.25 \\
 P(\text{yes}) &= P(\text{Yes}) * P(L|\text{Yes}) * P(\text{White}|\text{Yes}) = 4/8 * 0 * 0.25 = 0.025 \\
 P(\text{no}) &= P(\text{No}) * P(L|\text{No}) * P(\text{White}|\text{No}) = 4/8 * 1/4 * 1/4 = 1/32
 \end{aligned}$$

10. You are given the following information about a new car model. The name of the associated random variables is given in brackets.

- (Battery) The battery is fully charged with 60% probability.
- (Starter) The starter is working with 50% probability.
- (Fuel) The tank is empty with 50% probability.
- (Panel) If the battery is fully charged, the probability of the instrument panel working is 90%. If the battery is not fully charged, then the instrument panel will not work with 50% probability.
- (Fuel Gauge) If the tank is not empty and the instrument panel is working, then the fuel gauge will show empty with 10% probability. If the instrument panel is not working, then the fuel gauge will show empty with 100% probability. If the instrument panel is working and the tank is empty, the fuel gauge will show empty with 80% probability.

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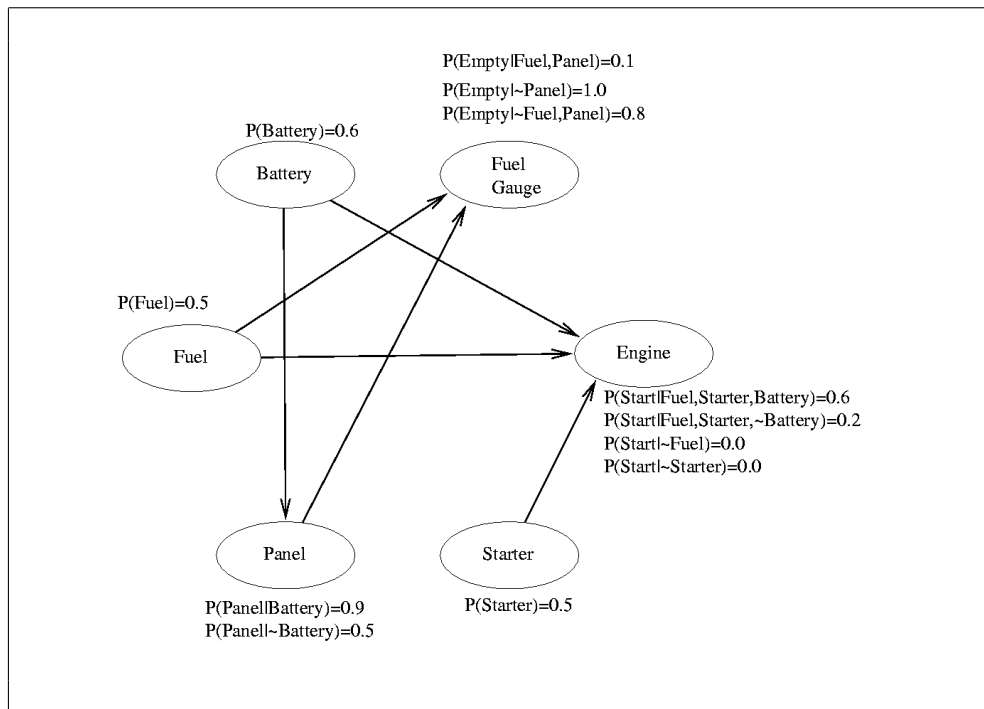
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- (Engine) If the tank is not empty, and the starter is working, and the battery is charged, the car will start with 60% probability. If the tank is not empty, and the starter is working, and the battery is not charged, then the car will start with 20% probability. In all other cases, the car will start with 0% probability.

Describe this information in the form of a bayesian belief network. Show the graph of the Bayesian belief network as well as all the conditional probabilities for all random variables.

[5 marks]



11. What is the probability of your car starting successfully if you try and start it. In other words, calculate $P(\text{Engine}=\text{Start})$.

[5 marks]

$$P(\text{Start}|\text{!Empty}, \text{Starter}, \text{Battery}) * P(\text{!Empty}) * P(\text{Starter}) * P(\text{Battery}) + P(\text{Start}|\text{!Empty}, \text{Starter}, \text{!Battery}) * P(\text{!Empty}) * P(\text{Starter}) * P(\text{!Battery}) = 0.6 * 0.5 * 0.5 * 0.6 + 0.2 * 0.5 * 0.5 * 0.4 = 0.11$$

12. Given that the fuel gauge shows that the tank is not empty, what is the probability of the car starting in this case. In other words, calculate $P(\text{Engine}=\text{Start} || \text{Fuel Gauge}=\text{not empty})$.

[5 marks]

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$$\begin{aligned}
&P(\text{Starter}, \text{Fuel}, \text{Battery}, \text{Panel}, \text{FG}, \text{Engine}) = \\
&P(\text{Starter}) * P(\text{Fuel}) * P(\text{Battery}) * P(\text{Panel} | \text{Battery}) * P(\text{FG} | \text{Fuel}, \text{Panel}) * P(\text{Engine} | \text{S}) \\
&0.5 * 0.5 * 0.6 * 0.9 * 0.9 * 0.6 = 0.073 \\
&+ \\
&P(\text{Starter}, \text{Fuel}, \sim\text{Battery}, \text{Panel}, \text{FG}, \text{Engine}) = \\
&P(\text{Starter}) * P(\text{Fuel}) * P(\sim\text{Battery}) * P(\text{Panel} | \sim\text{Battery}) * P(\text{FG} | \text{Fuel}, \text{Panel}) * P(\text{Engine} \\
&0.5 * 0.5 * 0.4 * 0.5 * 0.9 * 0.2 = 0.009 \\
&= 0.082 \\
&P(\sim\text{Empty}) = \\
&P(\sim\text{Empty} | \text{Fuel}, \text{Panel}) * P(\text{Panel} | \text{Battery}) * P(\text{Battery}) * P(\text{Fuel}) = \\
&0.9 * 0.9 * 0.6 * 0.5 = 0.2430 \\
&+ P(\sim\text{Empty} | \text{Fuel}, \text{Panel}) * P(\text{Panel} | \sim\text{Battery}) * P(\sim\text{Battery}) * P(\text{Fuel}) = \\
&0.9 * 0.5 * 0.4 * 0.5 = 0.09 \\
&+ P(\sim\text{Empty} | \sim\text{Fuel}, \text{Panel}) * P(\text{Panel} | \sim\text{Battery}) * P(\sim\text{Battery}) * P(\sim\text{Fuel}) = \\
&0.2 * 0.9 * 0.6 * 0.5 = 0.054 \\
&+ P(\sim\text{Empty} | \sim\text{Fuel}, \text{Panel}) * P(\text{Panel} | \sim\text{Battery}) * P(\sim\text{Battery}) * P(\sim\text{Fuel}) = \\
&0.2 * 0.5 * 0.4 * 0.5 = 0.02 \\
&= 0.4070 \\
&P(\text{Engine} | \sim\text{Empty}) = 0.20
\end{aligned}$$

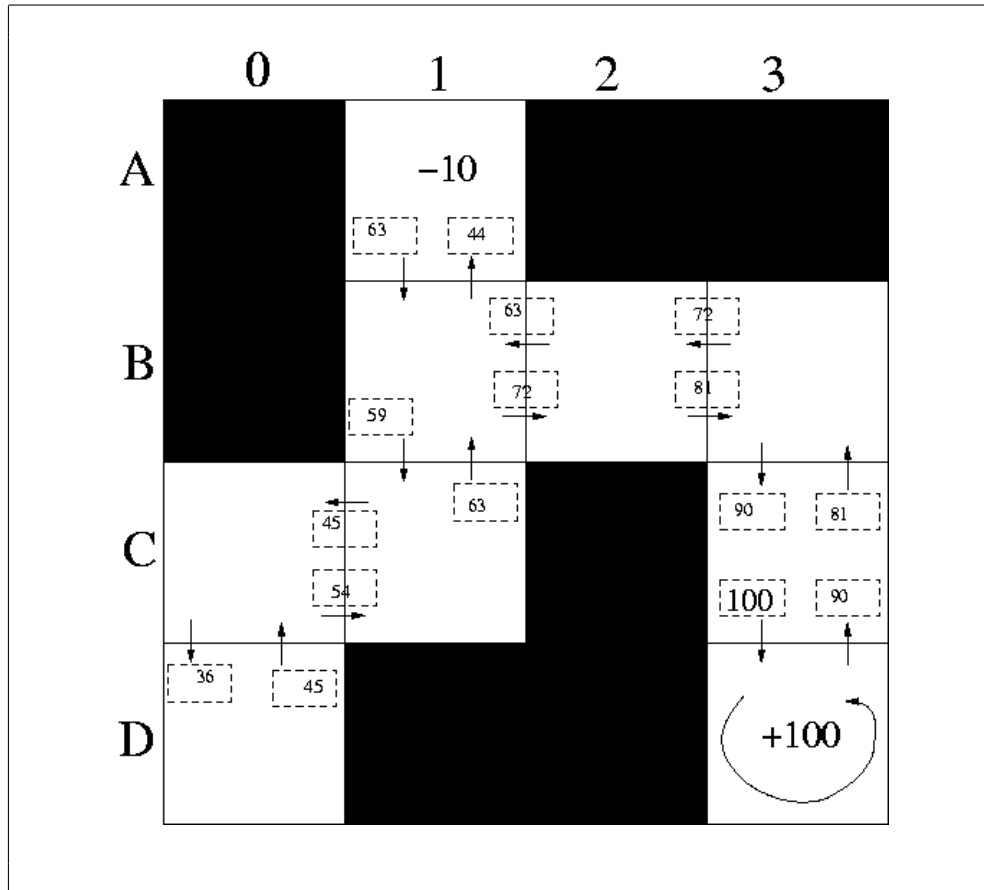
Section E: Reinforcement Learning

13. Below is a figure for a mobile robot environment. The robot T2 can move North, South, East and West only. The robot has a global positioning system; in other words, it knows which state $A0..D3$, it is in. The robot also includes a sonar sensor that can sense obstacles in all four directions. The actuators of the robot are very accurate; the robot will always move exactly one square, unless the target square is occupied by an obstacle, in which case the robot will not move. Once the robot reaches $D3$, the episode is terminated. The reward values are shown in the centre of the squares. Some of the Q-values are shown beside the action arrows. Fill in the missing Q-values, assuming that $\gamma = 0.9$. The policy π is the optimal policy, that is the robot will always choose the action with the best Q-value from the current state.

[5 marks]

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14. Assume that robot T2's (as described in question 13) global positioning system fails. Now the only feedback that T2 receives about its current state is from the sonar array. It can only sense obstacles (or the edge of the playing field) to the north, south, east, or west. Assume that T2 now learns the Q-values shown for the problem in question 13. Which Q-values will be different from the ones learned in question 13 (i.e., when T2's global positioning system was working). Show the new Q-values, assuming that T2 will visit each state with uniform probability.

[5 marks]

The following Q values will be different from the ones in question 13:(D0,D3).
 $Q(D0,N) = Q(D3,N) = 135/2=67.5$.

15. The online TD(λ) reinforcement learning algorithm using an ϵ -greedy policy is shown below.

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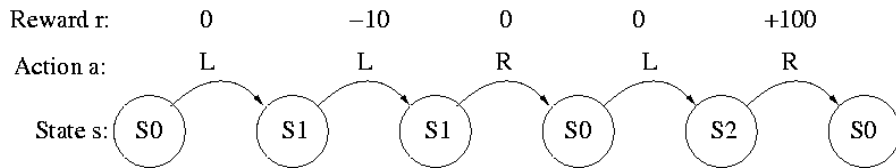
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 $\forall s, a \quad Q(s, a) := \text{Initialize to random value}$ 
do Repeat for each episode
   $\forall s, a \quad e(s, a) := 0$ 
   $s := \text{Start state}, a := \text{First action}$ 
  do Repeat for each state in the episode
    Take action  $a$ , observe  $r$ , and  $s'$ 
    Choose  $a'$  from  $s'$  using  $\epsilon$ -greedy policy
     $\delta := r + \gamma Q(s', a') - Q(s, a)$ 
     $e(s, a) := e(s, a) + 1$ 
    do  $\forall s, a$ 
       $Q(s, a) := Q(s, a) + \alpha \delta e(s, a)$ 
       $e(s, a) := \lambda \gamma e(s, a)$ 
    od
     $s := s', a := a'$ 
  od
od
    
```

Below is a trace of the TD(λ) algorithm for a short episode. Show the Q-table and eligibility values for each state action pair after the robot reaches the end of the sequence. The TD(λ) algorithm uses the following parameters: $\alpha = 0.3, \lambda = 0.6, \gamma = 0.7$.

If it is impossible to determine the Q values and eligibilities, then say so in your answer and explain why.



[10 marks]

Q-Table			
$Q(s, a)$	S0	S1	S2
L	12.66	-0.82	0.00
R	0.00	5.18	30.03

Eligibility			
$e(s, a)$	S0	S1	S2
L	0.189	0.031	0.000
R	0.000	0.074	0.420

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Additional work pages

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Additional work pages